平成30度前期 微分方程式 演習 18.05.02 番号 氏名 解答例

[1] 次の微分方程式を解け(変数分離形)

$$y' = \frac{y^2}{x}$$

[2] 次の微分方程式を解け(同次形)

$$x^2 y' = x^2 + y^2/4$$

[3] 次の微分方程式を解け(ベルヌーイ形)

$$y' - \frac{y}{x} = x^2 y^3$$

(変数分離形)

$y' = p(x)g(y)$ の形になるもの

$$\frac{dy}{dx} = p(x)g(y) \Rightarrow \int \frac{1}{g(y)}\,dy = \int p(x)\,dx$$

($\frac{1}{g(y)} y' = p(x)$ の両辺を x で積分

左辺は置換積分として見る)

(同次形)

$y' = g\left(\frac{y}{x}\right)$ の形に変形できるもの

$u = \frac{y}{x} \left(= \frac{y(x)}{x}\right)$ とおくと

$$y = xu,\quad y' = u + xu'$$

$$xu' = y' - u = g(u) - u$$

$$u' = \frac{1}{x}(g(u) - u)$$

(変数分離形)

(ベルヌーイ型)

$y' + p(x)y = g(x)y^n$ …① の形

$u = y^{1-n}$ とおくと

$$u' = y'(1-n)y^{-n}$$

① の両辺に $(1-n)y^{-n}$ をかけて

$$(1-n)y'y^{-n} + (1-n)p(x)y^{1-n} = (1-n)g(x)$$

$$u' + (1-n)p(x)u = (1-n)g(x)$$

(一階線形微分方程式,非斉次)

[1]

$$\frac{y'}{y^2} = \frac{1}{x}$$

$$\int \frac{1}{y^2}dy = \int \frac{1}{x}dy$$

$$-\frac{1}{y} = \log x + C$$

$$y = \frac{1}{-\log x - C} \quad (C\text{は定数})$$

[2]

$u = \frac{y}{x}$ とおくと $y = xu$

$$y' = u + xu',\quad xu' = y' - u$$

$$= \left(x^2 + \frac{y^2}{4}\right)\frac{1}{x^2} - u$$

$$= 1 - u + \frac{u^2}{4}$$

$$\frac{u'}{\left(\frac{u}{2}-1\right)^2} = \frac{1}{x}$$

$$\frac{2}{1-\frac{u}{2}} = \log x + C$$

$$2 - u = \frac{4}{\log x + C}$$

$$u = 2 - \frac{4}{\log x + C} \quad (C\text{は定数})$$

$$y = xu = 2x - \frac{4x}{\log x + C}$$

[3]

$u = y^{-2}$ とおくと $u' = y'(-2)y^{-3}$

$$\frac{y^3}{-2}u' - \frac{y^3}{x}u = x^2y^3$$

$$u' + 2\frac{u}{x} = -2x^2$$

$$u(x) = e^{-\int \frac{2}{x}dx}\left[\int -2x^2 e^{\int \frac{2}{x}dx}dx\right]$$

$$= \frac{1}{x^2}\left(\int -2x^4 dx\right)$$

$$= -\frac{2}{5}x^3 + \frac{C}{x^2} \quad (C\text{は定数})$$

$$y = u^{-\frac{1}{2}}$$

$$= \frac{1}{\sqrt{\frac{C}{x^2} - \frac{2}{5}x^3}} \quad (\ :\ )$$