演習解答例200527

2020 度前期 微分方程式 演習　20.05.27　学籍番号　　　　　　　　氏名

[1] 次の微分方程式を解け(変数分離形)

$$y' = \frac{y^2 - 1}{x}$$

[2] 次の微分方程式を解け(同次形)

$$x y' = x + y$$

[3] 次の微分方程式を解け(ベルヌーイ形)

$$y' + y = 2 x y^2$$

[1]

$$\frac{1}{y^2-1} y' = \frac{1}{x}$$

$$\int \frac{1}{y^2-1} dy = \int \frac{1}{x} dx$$

$$\int \frac{1}{2}\left(\frac{1}{y-1} - \frac{1}{y+1}\right) dy = \int \frac{1}{x} dx$$

$$\frac{1}{2} \log\left|\frac{y-1}{y+1}\right| = \log|x| + C$$

$$\log\left|\frac{y-1}{y+1}\right| = \log e^{2C}|x|^2$$

$$\frac{y-1}{y+1} = \pm e^{2C}|x|^2$$

$$y - 1 = C_1 (y+1)|x|^2$$

$$y(1 - C_1 x^2) = C_1 x^2 + 1$$

$$y = \frac{}{1 - C_1 x^2}$$

[2]

$$x y' = x + y$$

$$y' = 1 + \frac{y}{x}$$

$$u = \frac{y}{x}, \quad y = ux$$

$$y' = u'x + u$$

$$u'x + u = 1 + u$$

$$u' = \frac{1}{x}$$

$$u = \log|x| + C$$

$$\frac{y}{x} = \quad y = x\log|x| + Cx$$

[3]

$$y' + y = 2xy^2$$

$u = y^{1-2} = \frac{1}{y}$ とおく

$$-y^{-2}y' - y^{-1} = -2x$$

$$u' = -\frac{1}{y^2} y'$$

$$u' - u = -2x \quad \cdots ①$$

$u = ax + b$ とおくと

$$u' - u = a - ax - b = -2x$$

$$a = 2 \quad b = 2$$

$u = 2x + 2$ が ① の特解

斉次方程式の一般解は Ce^x

より $u = 2x + 2 + Ce^x$

(Cは定数)

$$y = \frac{1}{u} = \frac{1}{2x + 2 + Ce^x}$$