4/22 Notation ,Definition

Notations

$\mathbb{R}$: all real numbers $(-\infty, \infty)$

$[a, b] = \{x \mid a \leq x \leq b\}$ closed interval (compact)

$(a, b \in \mathbb{R},\ a < b)$

$f : [a, b] \to \mathbb{R}$: function defined on $[a, b]$

$\overset{def}{\iff}$ $\forall x \in [a, b]$, $\exists_1 f(x) \in \mathbb{R}$

(for any $x \in [a, b]$ one and only one real number $f(x)$ corresponds.)

f : (a function on $[a, b]$) is bounded (bdd in short.)

$\overset{def}{\iff}$ $\exists M > 0$ s.t. $|f(x)| \leq M$ $\forall x \in [a, b]$

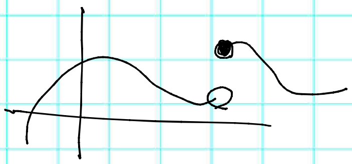
f is continuous (on $[a, b]$)

$\overset{def}{\iff}$ $\lim_{x \to p} f(x) = f(p)$ $\forall p \in [a, b]$

Property A continuous function on $[a, b]$ is bounded.

f : is right continuous and has left limit (RCLL)

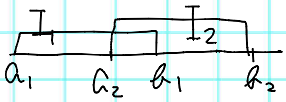
$\overset{def}{\iff}$ $f(p) = \lim_{x \downarrow p} f(x)$, $\exists \lim_{x \uparrow p} f(x)$



$I_1 = [a_1, b_1]$, $I_2 = [a_2, b_2]$

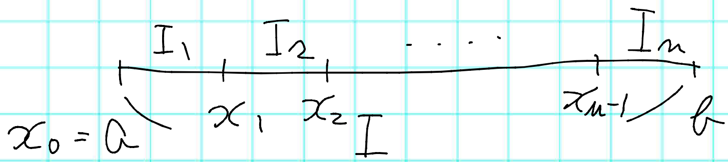
I_1 and I_2 are almost disjoint

$\overset{def}{\Longleftrightarrow}$ $a_1 < b_1 \leq a_2 < b_2$ or $a_2 < b_2 \leq a_1 < b_2$

( : Not almost disjoint)

$\{I_j\}_{j=1}^{n}$ is partition of I (finite set of closed intervals)

$\Longleftrightarrow$ $I = \bigcup_{j=1}^{n} I_j$, $\{I_j\}$: almost disjoint

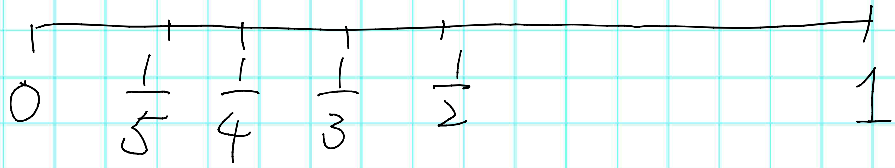


$\exists \{x_j\} = a = x_0 < \cdots < x_n = b$

$I = [a, b]$, $I_j = [x_{j-1}, x_j]$

<u>Example</u> $\{[0, \frac{1}{5}], [\frac{1}{5}, \frac{1}{4}], [\frac{1}{4}, \frac{1}{3}], [\frac{1}{3}, \frac{1}{2}], [\frac{1}{2}, 1]\}$

is a partition of $[0, 1]$



- Not always same lengths.

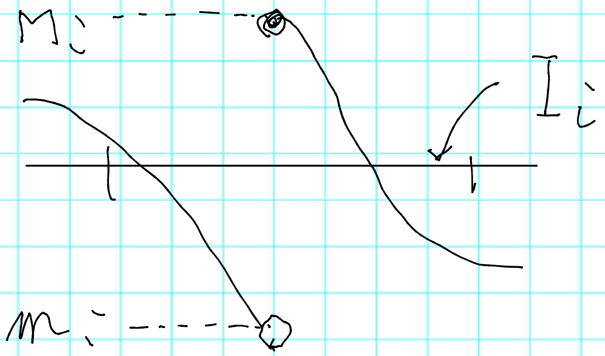
$|I|$ denotes the length of I. ($I = [a, b] \Rightarrow |I| = b - a$).

$|I| = \sum_{j=1}^{n} |I_j| \left(= \sum_{j=1}^{n} (x_j - x_{j-1}) = x_n - x_0 = b - a \right)$

$I=[a,b]$, $P=\{I_i\}$: partition of I.

f : function

$m_i = \inf\limits_{x\in I_i} f(x)$ $\quad$ $M_i = \sup\limits_{x\in I_i} f(x)$



$m = \inf\limits_{x\in I} f(x)$, $M = \sup\limits_{x\in I} f(x)$

$m \le m_i \le M_i \le M \quad \forall i \le n$

$L(f,P) = \sum\limits_{i=1}^{n} m_i |I_i|$, $U(f,P) = \sum\limits_{i=1}^{n} M_i |I_i|$

lower Riemann Sum $\qquad$ Upper Riemann Sum

Π : all partitions of I

$L(f) = \sup\limits_{P\in\Pi} L(f,P)$, $M(f) = \inf\limits_{P\in\Pi} M(f,P)$

f : Riemann integrable

$\underset{def}{\iff} \quad L(f) = M(f)$

$\int_a^b f(x)dx = L(f) = M(f)$