7/29 Application

Mean value : X : dice

$$\Pr(X=k) = \frac{1}{6} \quad k = 1, 2, \cdots, 6$$

$$f(x) = \begin{cases} \frac{1}{6} & x \in [0,1] \\ \vdots & \\ \frac{1}{6} & x \in (5,6] \end{cases}$$

$$g(x) = \begin{cases} 1 & x \in [0,1] \\ \vdots & \\ 6 & x \in (5,6] \end{cases}$$

$$\mathbb{E}[X] = \int_0^6 g(x) f(x)\, dx = 1 \times \frac{1}{6} + \cdots + 6 \times \frac{1}{6}$$

$$= 3.5$$

$\tilde{X}$: $\Pr(X=1) = \Pr(X=6) = \frac{1}{4}$

$\Pr(X=2) = \Pr(X=3) = \Pr(X=4) = \Pr(X=5) = \frac{1}{8}$

$$\tilde{f}(x) = \begin{cases} \frac{1}{4} & x \in [0,1] \cup (5,6] \\ \frac{1}{8} & x \in (2,5] \end{cases}$$

$$\mathbb{E}[\tilde{X}] = 1 \times \frac{1}{4} + 6 \times \frac{1}{4} + 2 \times \frac{1}{8} + \cdots + 5 \times \frac{1}{8} = 3.5$$

$$f^*(x) = \begin{cases} P_1 & x \in [0,1] \\ \vdots & \\ P_6 & x \in (5,6] \end{cases}$$

$$\Pr(X^* = k) = P_k$$

$$\mathbb{E}[X^*] = \int g(x) f^*(x)\, dx$$

$$\Pr(Y \leq r) = \int_0^r f(x)\, dx \Rightarrow \mathbb{E}[Y] = \int_0^M x f(x)\, dx$$

$$\int_0^M f(x)\, dx = 1$$

◎ inner product

$$\vec{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_d \end{pmatrix} \quad \vec{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_d \end{pmatrix} \qquad \vec{x}\cdot\vec{y} = \sum_{k=1}^{d} x_k y_k$$

$f, g : [0,1] \to \mathbb{R}$ conti,

$$(f, g) = \int_0^1 f(x)\, g(x)\, dx$$

$$\vec{x}\cdot(a\vec{y}_1 + b\vec{y}_2) = a\,\vec{x}\cdot\vec{y}_1 + b\,\vec{x}\cdot\vec{y}_2$$

$$(f, ag_1 + ag_2) = a(f, g_1) + b(f_1, g_2)$$

$$\vec{e}_k = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \subset k \qquad \vec{e}_k\cdot\vec{e}_j = \delta_{k,j}.$$

$$\vec{x} = \sum_{k=1}^{d} (\vec{x}\cdot\vec{e}_k)\,\vec{e}_k$$

$$\exists \{\varphi_k\}_{k=1}^{\infty} \qquad \int \varphi_k(x)\,\varphi_j(x)\,dx = \delta_{k,j}.$$

$$f(x) = \sum_{k=1}^{\infty} (f, \varphi_k)\,\varphi_k$$

Fourier Expansion

$$\begin{cases} a_n = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(t)\cos nt\, dt \\ b_n = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(t)\sin nt\, dt \\ f(t) = \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty}(a_n \cos nt + b_n \sin nt) \end{cases}$$

Fourier transform

$f : \mathbb{R} \to \mathbb{R}$

$$F(t) = \int_{-\infty}^{\infty} f(x) e^{-itx} dx$$

$$\Rightarrow f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(t) e^{itx} dx$$

$$\gamma_\sigma(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

$$\Gamma_\sigma(t) = e^{-\frac{\sigma^2 t^2}{2}} \left(= \int \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} e^{-itx} dx \right)$$

$$\gamma_\sigma(x) \xrightarrow[\sigma \to 0]{} \delta_0(x), \quad \Gamma_\sigma \xrightarrow[\sigma \to 0]{} 1$$